MATHEMATICS ADMISSIONS TEST

For candidates applying for Mathematics, Computer Science, or one of their joint degrees at the UNIVERSITY OF OXFORD.

October 2025

Time allowed: $2\frac{1}{2}$ hours

In 2025, the MAT was administered by Pearson VUE, and candidates answered the questions electronically. There were 25 multiple-choice questions and two longer questions with multiple parts, for which candidates typed answers.

The multiple-choice questions (Q1 – Q25) will not be released.

The long questions (Q26 and Q27) are reproduced below.
This year, candidates were shown either version X or version Y of the long questions (labelled below). MAT scores were viewed with the added information of whether candidates were shown version X or version Y.

Question 26 X

This question is about two diagrams.

In parts (i) and (ii) and (iii), we consider a diagram formed of eight points equally spaced around the unit circle, with each pair of points connected with a straight line segment, except for cases where the two points lie directly opposite each other.

In parts (iv) and (v), we will consider a new diagram formed of three separate collections of points named A, B, and C respectively.
A contains 3 points, B contains 4 points, and C contains 5 points.
Each point in A is connected with a straight line segment to each point in B.
Each point in B is connected with a straight line segment to each point in C.
Each point in C is connected with a straight line segment to each point in A.
No other pairs of points are connected with a straight line segment.

(i) How many line segments are there in the first diagram in total? Explain your answer.

2 marks

(ii) A 3-*star* is a collection of three distinct points, with each pair connected by a straight line segment. How many 3-stars are there in the first diagram in total? Explain your answer. **3 marks**

(iii) For $n \geq 4$, an n-*star* is a collection of n distinct points, with each pair connected by a straight line segment. For $n = 4, 5, 6, 7, 8$, how many n-stars are there in the first diagram in total? Explain your answers. **3 marks**

(iv) A new diagram is formed of three separate collections of points named A, B, and C respectively. A contains 3 points, B contains 4 points, and C contains 5 points.

Each point in A is connected with a straight line segment to each point in B.

Each point in B is connected with a straight line segment to each point in C.

Each point in C is connected with a straight line segment to each point in A.

No other pairs of points are connected with a straight line segment.

How many 3-stars are there in this new diagram in total? Justify your answer carefully. **3 marks**

(v) For $n \geq 2$, an n-*loop* is a sequence of n distinct points with each point connected to the next in sequence with a straight line segment, and with the last connected to the first with a straight line segment.

For which values of $n \geq 4$ does the new diagram described in part (iv) contain at least one n-loop? Justify your answer. **4 marks**

Question 26 Y

Linear polynomials are functions of the form $a + bx$ with a and b real numbers.

Given two linear polynomials, we define an operation $\cdot$ as follows. To calculate $f(x) \cdot g(x)$, we multiply the polynomials as normal, but we ignore any x^2 term.

For example, $(1 - 2x) \cdot (3 - 5x) = 3 - 11x$, because we are ignoring the $10x^2$ term.

(i) (a) Find the coefficients of $(a+bx) \cdot (c+dx)$ in terms of a, b, c, d, and explain why $f(x) \cdot g(x) = g(x) \cdot f(x)$ for all linear polynomials $f(x)$ and $g(x)$.

(b) Find a linear polynomial $f(x)$ such that $f(x) \cdot (1 + 2x) = (3 + 4x)$.

3 marks

(ii) A strictly increasing linear function $f(x)$ has the property that if $y > x$ then $f(y) > f(x)$.

A student claims that if $f(x)$ and $g(x)$ are both strictly increasing linear functions, then so is $f(x) \cdot g(x)$. Is the student correct? If so, prove the student's claim. Otherwise, find a counterexample.

2 marks

(iii) Prove that

$$(f(x) \cdot g(x)) \cdot h(x) = f(x) \cdot (g(x) \cdot h(x))$$

for all linear polynomials $f(x)$ and $g(x)$ and $h(x)$.

[For the rest of the question, we will assume this result and write $f(x) \cdot g(x) \cdot h(x)$ to mean either expression above.]

2 marks

(iv) Let N be a whole number with $N > 1$. Find

$$(1 + x) \cdot (1 + 2x) \cdot (1 + 3x) \cdot (1 + 4x) \cdot \ldots \cdot (1 + 2Nx).$$

3 marks

(v) Given that $f(x)$ and $g(x)$ are linear polynomials and $f(x) \cdot g(x) = 0$, describe all possibilities for the pair $f(x)$ and $g(x)$. **2 marks**

(vi) Given that $f(x)$ and $g(x)$ and $h(x)$ are linear polynomials and

$$f(x) \cdot g(x) \cdot h(x) = 0,$$

prove that at least one of the following statements must be true;

(I) $f(x) \cdot g(x) = 0$,

(II) $g(x) \cdot h(x) = 0$,

(III) $f(x) \cdot h(x) = 0$.

For each of the three statements, give examples of polynomials for which that statement is true and the other two statements are false. **3 marks**

Question 27 X

In this question we will write $\{1, 2, 4, 5\}$ to mean the set containing the numbers 1, 2, 4, and 5. The order of the numbers inside a set doesn't matter, and the numbers in a set cannot be repeated. All the sets in this question contain whole numbers (positive, or zero, or negative).

We say that a set of whole numbers is nice if there is a number T such that the numbers in the set can be paired together so that each pair adds up to T. If such a number T exists, we will call T a *target* for that set.

For example, the set $\{1, 2, 5, 6\}$ is nice because we can take the target to be $T = 7$ and pair the elements as $1 + 6 = 2 + 5$. The set $\{1, 3, 4, 5\}$ is not nice because there is no such number T.

In the second half of this question, we will imagine that each whole number is coloured either red or green, and we will look for nice sets with all numbers the same colour.

(i) Is the set $\{1, 2, 4, 5, 6, 9, 10, 11\}$ nice? Justify your answer. **2 marks**

(ii) Prove that for any nice set of six numbers, the total of those six numbers must be a multiple of 3. **1 mark**

(iii) Now we consider infinite sets of whole numbers (for example, the set of all the positive whole numbers).

Give examples to demonstrate that an infinitely large set of whole numbers might have zero, exactly one, or more than one target(s).

Justify your answers, making it clear which is which. **6 marks**

(iv) Suppose that we want to find a nice set of two numbers that are the same colour. By considering the possibilities for the colours of the numbers 1, 2, and 3, prove such a set always exists. **1 mark**

(v) Suppose that we want to find a nice set of four numbers that are all the same colour. By considering the possibilities for the colours of the numbers from 1 to 27 inclusive, prove such a set always exists.

[Hint: consider $\{1, 2, 3\}$, $\{4, 5, 6\}$, ..., $\{25, 26, 27\}$.] **5 marks**

Question 27 Y

An office building has n meeting rooms numbered 1, 2, ..., n according to their size. Meeting room 1 only has space for one person, meeting room 2 has space for two people, and so on, up to meeting room n, which has space for n people.

The office has n teams, numbered 1, 2, ..., n who call reception in that order one-by-one to book a meeting room. For $i = 1, \ldots, n$, team i has size $t(i)$, where $t(i)$ is a whole number with $1 \leq t(i) \leq n$. When team i calls reception, they are given the *smallest* available meeting room among meeting rooms with size greater than or equal to $t(i)$. If all these rooms have already been booked, then team i do not manage to book a meeting room.

We represent the sizes of the teams using a list $(t(1), t(2), \ldots, t(n))$. We call the list *good* if all teams manage to book a meeting room, and we write $G(n)$ for the number of good lists of length n.

For example with $n = 2$, each of the lists $(1, 1)$, $(1, 2)$, $(2, 1)$ is good because for each of them both teams manage to make a booking, whereas the list $(2, 2)$ is not good because the first team takes meeting room 2 and then the second team doesn't fit in the remaining meeting room. We have $G(2) = 3$.

From part (iii) onwards, we will define $F(n, k)$ to be the number of good lists of length n which result in team n booking room k.

(i) For $n = 3$, explain why the list $(2, 1, 1)$ is good, but the list $(2, 2, 2)$ is not good. **2 marks**

(ii) For $n = 3$, there are 16 good lists, so $G(3) = 16$. List all of them, starting with good lists with $t(1) = 1$, then good lists with $t(1) = 2$, and then good lists with $t(1) = 3$. **3 marks**

(iii) For $k = 1, \ldots, n$, let $F(n, k)$ be the number of good lists of length n which result in team n booking room k.

Explain why $F(n, k)$ is a multiple of k. **2 marks**

(iv) Describe the relationship between $G(n)$ and $F(n, 1)$, $F(n, 2)$, ..., $F(n, n)$. **1 mark**

(v) Explain why $F(4, 1) = G(3)$ and $F(4, 4) = 4 \times G(3)$. **3 marks**

(vi) Find the values of $F(4, 2)$ and $F(4, 3)$. Explain your answers in each case. Hence find the value of $G(4)$. **4 marks**